ment with the distorted-wave calculation of Davison[19] who used the same potential as that given in Eq. (6.1). Davison, however, took proper account of the identity of the two hydrogen molecules while in the present research a distinction is made between the incident and target hydrogen molecules. For this reason, the cross sections could not be expected to agree exactly.

We can now compute the probability for rotational de-excitation from the $j=2$ state upon collision in H_2 gas and compare this with the value determined by ultrasonic dispersion measurements.[20] This probability for rotational de-excitation can be obtained from the following expression.

$$P_{20}=\sigma(0\leftarrow 2)/\sigma_{\text{kin}}(0\leftarrow 0) \qquad (6.2)$$

where $\sigma_{\text{kin}}(0\leftarrow 0)$ is the elastic scattering cross section between H_2 molecules as measured in kinetic theory experiments. According to reference 20 $\sigma_{\text{kin}}(0\leftarrow 0)=23.2$ Å^2 in H_2 gas. To be strictly correct, we should average the curve in Fig. 6 over a Maxwell-Boltzmann distribution of incident energies before dividing by $\sigma_{\text{kin}}(0\leftarrow 0)$, but to simplify matters we forego the averaging procedure and use the value of $\sigma(0\leftarrow 2)$ at $E_{\text{inc}}=kT$. For $T=300$°K, $kT=0.0259$ eV, and $\sigma(0\leftarrow 2)$ at $E_{\text{inc}}=0.0259$ eV is seen to have the value 0.020 from Table V. Therefore,

$$\text{for } T=300°\text{K},\quad P_{20}\approx 0.020/23.2, \approx 0.86\times 10^{-3}. \qquad (6.3)$$

The experimental value for P_{20} as given in reference 20 is

$$\text{for}\quad T=300°\text{K},\quad P_{20}=3.0\times 10^{-3} \text{ (experimental)}. \qquad (6.4)$$

The fact that the computed de-excitation probability is low may be due to the fact that we have not considered processes where both molecules come together in the $j=2$ state and one is de-excited to the $j=0$ state while the other is excited to the $j=4$ state, or to the fact that the value of β in the potential (6.1c) is too small. Since the inelastic cross section goes approximately as β^2, a value of $\beta=0.14$ will produce the correct answer. This is in exact agreement with the conclusion reached by Davison[19] in his calculation. Since it is not clear that Eq. (6.1) does indeed represent the correct potential to use for the averaged H_2-H_2 interaction potential, it is not unreasonable to surmise that perhaps β should actually be made larger.

ACKNOWLEDGMENTS

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Čerenkov-Like Radiation by Plasma Oscillations*

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A coupling mechanism is shown to exist between electrostatic oscillations along a plasma cylinder and transverse electromagnetic waves outside, resulting in radiation by phase oscillations. Plasma oscillations along the cylinder can be decomposed into traveling waves with plasma velocity $v_p=\omega/k$. The macroscopic appearance of such a traveling wave is the same as if a series of alternating positive and negative charged bunches move with the phase velocity of the wave. For long wavelengths, where the phase velocity exceeds the speed of light in the surrounding medium, radiation takes place, having a pattern as expected from Čerenkov radiation of the charged bunches moving with v_p. Since for plasma oscillations $\omega/k>c$ modes exist (Dawson and Oberman), this radiation can also take place in vacuum.

A COLD electron plasma is known to exhibit electrostatic oscillations with the characteristic plasma frequency $\omega=\omega_p$. These oscillations are longitudinal, with the particle motion and electric field both parallel to the wave vector **k**. The phase velocity $v_p=\omega_p/k$ can take on any value; in fact, it can exceed the speed of light in vacuum if the wave number is small enough: $k<\omega_p/c$.

This electrostatic plasma wave consists macroscopically of a sinusoidal spatial variation of space charge traveling with the velocity v_p. This is, of course, brought about by electrons exhibiting small oscillations about their equilibrium positions with their phases properly adjusted to give the above described macroscopic space charge wave.

In an infinite uniform plasma, longitudinal plasma oscillations have the interesting property that the conduction and displacement currents cancel exactly.

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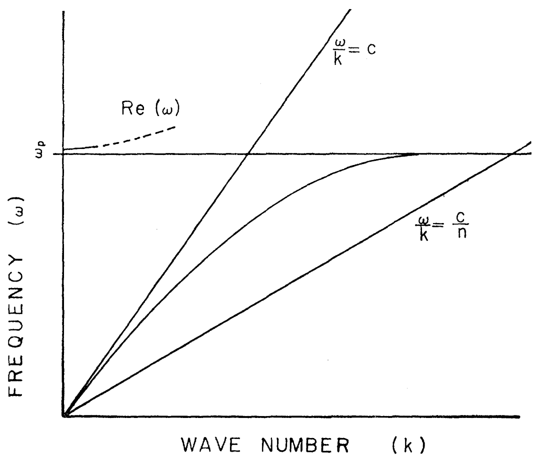

FIG. 1. Shape of the dispersion curve for electron plasma oscillations along a plasma slab or cylinder in a strong magnetic field ($\mathbf{B}_0 \| \mathbf{k}$).

Hence, no magnetic field and, thus, no radiation is associated with these oscillations. Consider now longitudinal plasma oscillations along the axis of a plasma cylinder of finite radius R. If, by choosing k small enough, the phase velocity can be made to exceed the speed of light in vacuum, one expects the appearance of Čerenkov-like radiation from the "fast moving" charge bunches. If the plasma cylinder is embedded into a dielectric, radiation should also appear when the phase velocity is greater than the speed of light in the medium.

Consider first an infinite linear current source

$$\mathbf{j}=\mathbf{j}_0 \exp[i(kz-\omega t)]\delta(x)\delta(y),$$

with $\mathbf{j}_0$ in the z direction. The vector potential can be calculated from

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}(\mathbf{r})e^{-i\omega t},$$

where

$$\mathbf{A}(\mathbf{r})=\frac{\mu_0}{4\pi}\int \mathbf{j}(\mathbf{r}')\frac{\exp(ik_0|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}d^3r'$$

and $k_0 \equiv \omega/c$. This integral can be evaluated to yield

$$\mathbf{A}(\mathbf{r})=(\mu_0/2\pi)\mathbf{j}_0 e^{ikz}K_0[r(k^2-k_0^2)^{1/2}],$$

where r is the perpendicular distance from the z axis to the observation point. $K_0(x)=(\frac{1}{2}\pi i)H_0^{(1)}(ix)$ is the modified Bessel function of the second kind of zero order. For large values of the argument ($x \gg 1$), $K_0(x) \sim (\pi/2x)^{1/2}e^{-x}$. If the phase velocity of the source term is smaller than c, then $k_0 < k$ and the vector potential decays exponentially away from the source. If, on the other hand, the phase velocity exceeds c, then $k_0 > k$, and far away from the z axis one finds

$$\mathbf{A}(\mathbf{r},t) \sim \frac{\mu_0 \mathbf{j}_0}{(8\pi)^{1/2}} \exp[i(kz-\omega t)] \frac{\exp[-i(k_0^2-k^2)^{1/2}r]}{[ir(k_0^2-k^2)^{1/2}]^{1/2}}.$$

In this case $\mathbf{A}$, $\mathbf{E}$, and $\mathbf{B}$ vary with $r^{-1/2}$ far away, hence, the Poynting vector varies with $1/r$ as expected for radiation. From $\mathbf{A}$ one calculates $\mathbf{B}$ and $\mathbf{E}$. $\mathbf{B}$ has only a θ component while E has both r and z components; $E_z/E_r=(v_p^2/c^2-1)^{1/2}=\tan\theta_c$, where θ_c is the Čerenkov angle. For a standing wave the radiation field is just the superposition of the fields of two oppositely moving traveling waves.

The power radiated per unit length can be calculated:

$$P=\frac{\mu_0 j_0^2 \omega}{8}\left(1-\frac{c^2}{v_p^2}\right).$$

For $j_0=1$ A and $\omega=10^{10}$ sec^{-1} the radiated power is of the order of 1 kW/m.

In a thin cylindrical plasma, however,[1] $\omega \ll \omega_p$ and $v_p \ll c$. We consider, then, the radiation field of a finite cylinder of radius R with a uniform oscillating current density. The radiation field at large distances can be obtained by using the previous solution as a Green's function. One finds

$$\mathbf{A}=2\mathbf{A}_l J_1[(k_0^2-k^2)^{1/2}R]/(k_0^2-k^2)^{1/2}R,$$

where $\mathbf{A}_l$ is the solution from the linear source term, and J_1 the first-order Bessel function. Due to the destructive interference, the radiation field decreases with increasing R.

While phase velocities exceeding c can exist in an infinite plasma, the existence of such modes in a finite plasma cylinder is by no means obvious. Dawson and Oberman[2] investigated the dispersion relations in a plasma slab and cylinder situated in a strong static axial magnetic field. They set up the equations for the electromagnetic fields inside the plasma and outside in free space. They found plasma modes with $\omega/k > c$, and also discovered that the boundary conditions at the plasma-vacuum interfaces could be satisfied only if outgoing electromagnetic waves were present for these modes. These waves are identical with our Čerenkov radiation fields.

In a plasma slab or cylinder in a strong magnetic field, the dispersion relation splits into two branches[2] as shown in Fig. 1. In the lower branch $\omega/k < c$, while the upper branch is radiative, hence, the plasma oscillations are damped and ω is complex. The imaginary part of ω grows rapidly with increasing k, hence, in the dotted portion of the curve the oscillations damp out rapidly.

If the plasma column or slab is surrounded by a dielectric material (this can be a magnetoplasma itself) the lower branch also becomes radiative in the region where $\omega/k > c/n$, n being the index of refraction. Hence, such a plasma, if excited to oscillate, is bound to radiate in all of its longer wavelength modes. If n is large and the plasma diameter greatly exceeds the wavelength, the frequency of the radiation is approximately equal to the plasma frequency.

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